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Some Current and Proposed  
Theoretical Studies of Gravity at the  
University of Maryland

UNPUBLISHED PRELIMINARY DATA

C. W. Misner  
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## B. THEORETICAL RESEARCH

The current research of Drs. Misner and Zepolsky is concerned with theoretical models of terminal stages of stellar evolution where gravitational radiation might be a dominant feature, such as the collapse of a supernova core. The most intense gravitational radiation can be emitted only as general relativity begins to be important, so one emphasis in research here is on the description of highly condensed bodies involving strong gravitational fields. Intense gravitational radiation also requires a rotating mass which implies axial symmetry (rather than spherical) at the simplest. Dr. Zipoy's theoretical researches are providing insight into the special features of strong gravitational fields with this reduced symmetry. A second important part of the research program is a study of the way non-linearities of the gravitational field reduce the output of gravitational radiation below the linear theory predictions and determine the maximum in power output and frequency for, say, a binary collapsed star system resulting from rotationally induced fission of a collapsing supernova core. Various specific research problems in these areas are described below. Several are in preliminary stages, some are nearing completion.

### I. Maximum Static Equilibrium Cold Mass. (Misner, Zepolsky)

Many calculations have indicated<sup>1</sup> that large masses cannot exist in static equilibrium once nuclear energy sources are exhausted. The most recent calculations<sup>2</sup> which allow for pressure induced nuclear reactions (inverse  $\beta$  decay) and nuclear forces in some detail give a maximum around 1.5 sun's masses. However, elementary particle theory is not in a position to provide a reliable equation of state at densities in excess of nuclear density. By studying numerically some models with extreme high and low compressibilities at ultranuclear density, and by analytical analysis of a large family of other possibilities, one sees that it is sufficient for a calculation of the maximum mass to know the equation of state at nuclear density and below. The higher density behaviour of the equation of state has no influence on the maximum mass<sup>3</sup>. Thus one expects that for massive stars (5 - 50 sun's masses) the evolutionary processes will leave cores or fragments exceeding the maximum mass for cold equilibrium, and the final state must be one of continuing collapse<sup>4</sup>. Research in progress described below is directed to a more detailed study of this collapse process.

### II. Stability of Cold Massive Equilibria (Zepolsky)

All the known static equilibrium solutions of the Einstein equations (cf. I above) will be stable<sup>5</sup>. Recently Chandrasekhar<sup>6</sup> has given a method for demonstrating

instability in some cases. Suitably modified to allow for relativistic equations of state, it is being applied to test the stability of the ultra-density models previously constructed.

### III. Dynamics of Spherical Collapse (Misner, in collaboration with S. Colgate (Livermore), D. Sharp (Princeton), R. Lindquist (Adelphi University))

The Oppenheimer-Snyder<sup>4</sup> metric describes the spherically symmetric collapse of a uniform density sphere of matter with the simple equation of state  $p = 0$ , but including the effects of general relativity. These effects are quite dramatic, for just as the rapidly collapsing sphere approaches the characteristic Schwarzschild radius for the object (3 kilometers per solar mass), the gravitational red shift suddenly comes strongly into play and effectively brings the collapse motion (as seen by a distant observer) to a halt. Thus this continuing collapse can be regarded as a final (apparent equilibrium) state for a cold star whose mass exceeds the limit for true equilibrium. It is of great interest at present, both for questions of gravitational radiation and in connection with the quasi-stellar radio sources, to create more realistic theoretical models of continuing collapse. The simplest, and therefore first, problems in this connection retain the simplification of spherical symmetry and absence of rotation, but will allow for non-zero pressure in the equation of state, and possible mass loss through neutrino radiation or blowing-off of an outer shell of matter. The hydrodynamic codes and computational facilities at Livermore will be available for this project. The analytical work of formulating the problem and choosing interesting and meaningful output data is beginning now with the aim of having some simple problem already run when full time work on the project begins at Livermore in the summer.

### IV. Computing Through the Schwarzschild Singularity. (Hord and Misner)

One specific difficulty which any calculation of relativistic collapse must circumvent is the Schwarzschild singularity. In the process of collapse one expects a characteristic surface (from which there is a total redshift) to form which the matter will then fall through. The standard coordinates of the Schwarzschild metric become singular at this surface in a way that has only recently become well understood<sup>7</sup>. Of various analytical techniques which have been used to avoid using singular coordinates<sup>7,8</sup> it is not clear which if any is best suited to numerical computations. In order to study just this problem without the complications of a dynamic collapse, Hord has chosen to study a static problem (the static spherically symmetric solutions of coupled Einstein and Yukawa equations, i.e. a mass coupled to a scalar field) which cannot be solved analytically, and develop a method for exhibiting numerically the non-singular physical phenomena associated with the "Schwarzschild singularity" in this example.

## V. Static Solutions with Axial Symmetry (Zipoy)

It is expected that angular momentum will be an important feature in problems of catastrophic collapse, and that centrifugal forces could be more important than pressure in determining the collapse rate. But the rotation axis breaks spherical symmetry so the simplest of these problems must already be posed in the context of merely axial symmetry. To obtain some familiarity with relativistic geometry of this symmetry, a detailed study of some examples of Weyl's<sup>9</sup> solutions of the Einstein equations is in progress. They have been recast in oblate and prolate spheroidal coordinates as appropriate, and can be classified according to a multipole order of the "singularity". Some of the phenomena already studied in these metrics suggest that in a non-spherical collapse, the redshift could become total at the poles before the equator so that the object would more resemble a donut than a disk. Although such phenomena would be unobservable even if they occurred, they are of importance in designing (eventually) computing programs for more realistic collapse models. The computer must be prepared to cope with such behaviour in the cores of massive objects.

## VI. Football, Discs, Rings. (Hernandez)

Although the solutions of the empty space Einstein equations which Dr. Zipoy is studying are presumed to be the gravitational fields which might surround masses of ellipsoidal or ring shapes, this fact has not yet been established by exhibiting matching interior solutions covering the region occupied by the central mass. Mr. Hernandez is constructing solutions for these interior regions and will try to discover whether minimal requirements on the properties of the matter involved (such as finite stresses, positive energy density) will exclude large masses of high density, as is the case in spherical symmetry<sup>10</sup>. Since a fluid in static equilibrium will assume a spherical shape, the matter in question here must be assumed solid. The techniques used here should also lead to solutions showing solid rotating objects as (non-singular, simply interpretable) sources for the exterior gravitational field discovered recently by Kerr<sup>11</sup>. This would give a much better foundation to the assumed interpretation of the Kerr metric as the field produced by a rotating mass.

## VII. Non-Spherical Free Collapse (Hernandez and Misner)

It is hoped that the studies described above of spherical collapse (III) and axial geometry (V and VI) will provide a foundation from which it will be possible to make further progress on more realistic collapsing star models. Although angular momentum will be all important in connection with gravitational radiation, the first step beyond spherical collapse must ignore angular momentum and take the simplest dynamic

problems which can be posed involving axial symmetry. If it proves to be any simplification, the equation of state  $p = 0$  will also be invoked. Numerical solutions of the Einstein equations have been obtained previously under conditions of axial symmetry<sup>12</sup>, but they involved no relativistic hydrodynamics, and lacked a formulation of significant output parameters. The effort involved in incorporating the hydrodynamics may be well repaid by providing a simpler physical interpretation of the solution which will suggest useful output data.

#### VIII. Stationary Equilibria for Rotating Fluids (Schulsky)

In spherically symmetric situations, the Poisson equation  $\nabla^2 \phi = 4\pi G\rho$  for the generation of the gravitational field, and Newton's law for the force balance in equilibrium can be combined in a single equation of hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{G\rho}{r^2} \int_0^r 4\pi r'^2 dr'$$

The corresponding reduction of the relativistic equations of hydrodynamics and Einstein field equations to a single equation for spherically symmetric static equilibrium was given by Oppenheimer and Volkoff<sup>1</sup>, yielding

$$\frac{dp}{dr} = \frac{-G}{c^2} \frac{(\rho + p)}{r^2} \frac{(M + 4\pi \rho c^{-2} r^3)}{1 - (2M/r)}$$

where

$$M(r) = \frac{1}{c^2} \int_0^r \rho c 4\pi r'^2 dr'$$

Again in the Newtonian case, Euler's hydrodynamic equations and the Poisson gravitational field equation can be combined, in the case of a fluid rotating as a rigid body, to a single partial differential equation

$$4\pi G\rho = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \left[ \omega^2 r \sin^2 \theta - \frac{1}{\rho} \frac{\partial \rho}{\partial r} \right] + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \left[ \omega^2 r \sin \theta \cos \theta - \frac{1}{\rho} \frac{1}{r} \frac{\partial \rho}{\partial \theta} \right]$$

Work is now in progress to reduce the Einstein equations for this problem to an analogous form. This will be followed by analytical studies of the main properties of the relativistic equation. Numerical solutions will show whether a mass which exceeds the maximum possible in static equilibrium may, with sufficient angular momentum, have a stationary rotating equilibrium. These models will also provide starting points for eventual investigations of rotational collapse.

### IX. Gravitational Radiation--Redshift Cut-Off (Eichelstein)

Once collapse begins in a rotating mass as nuclear energy sources become exhausted, it is unlikely that axial symmetry will be maintained. With conservation of angular momentum, the angular velocity will increase as the size decreases, and one knows<sup>13</sup> that rapidly rotating fluid masses are more stable in non-symmetric forms (Jacobi ellipsoids) than in axially symmetric spheroidal forms. The ellipsoid could then fission to produce a binary system whose components would separately collapse further if their masses each exceed the maximum for cold equilibrium. There is little prospect that detailed dynamic models of the loss of axial symmetry or of the fission process can be created. But by assuming that angular momentum is conserved and that the final state is two nearly point masses in a binary system, one discovers that few stars seem to have sufficient angular momentum to prevent gravitational radiation from becoming important should they collapse in this way. Once a binary system begins emitting gravitational waves strongly, the power and frequency of the radiation increase while the orbit tightens due to the energy loss. The well known linearized theory calculations<sup>14</sup> predict no maximum to the power emitted, but some limit certainly exists below the natural gravitational unit of power  $c^5/G = 3.6 \times 10^{59}$  erg/sec. It is clearly a problem of primary interest for the design of gravitational observatories to know as precisely as possible just what the maximum power and maximum frequency are. This maximum occurs as non-linearities in the gravitational field equations become dominant, so any presently feasible calculation must find some special case or approximation to analyze. The special case Mr. Eichelstein is studying is a binary system where  $m_1 \gg m_2$ . In this case the motion of the second mass is essentially a geodesic in the Schwarzschild field produced by the larger mass. In the next approximation, the system will radiate, and the radiation can be calculated by solving an inhomogeneous linear partial differential equation with coefficients which depend only on  $r$ . (A similar calculation for electromagnetic radiation from a charge in a gravitational orbit has been carried out by Suna<sup>15</sup>, and some properties of the tensor wave equation in Schwarzschild space have been studied by Regge and Wheeler<sup>16</sup>.) The main result of this calculation will be a description of the way the frequency and power of the radiation are reduced by redshift effects as the orbit falls deep into the gravitational potential well.

### X. Gravitational Radiation--Terminal Phase in Radiative Collapse of a Binary System. (Vishveshwara)

Another special case for the computation of gravitational radiation outside the realm of linearized theory is a phase in the evolution of a pair of totally col-

lapsed stars long after the phase of maximum power output. It will describe the final approach of the system to a single, non-radiating, axially symmetric collapsed object. It may suggest limits on the fraction of the rest mass of the initial system which can be converted into gravitational radiation. It will provide some, perhaps limited, information about the power output peak by allowing one to try to extrapolate back to the peak from the final behaviour as one tries to extrapolate ahead to the peak from the initial behaviour in the linearized theory realm. The approximation which will make this calculation feasible will be an assumption that the system is little different from the Oppenheimer-Snyder<sup>4</sup> totally collapsed objects. The way in which these small differences should be described has been outlined before<sup>16</sup>, and some of the methods needed to calculate radiation from such a perturbation in the gravitational field are given by Hartle<sup>17</sup>. The gravitational fields for binary systems given by Brill and Lindquist<sup>18</sup> suggest possible configurations for a highly collapsed binary system.

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9. See Bergmann, "Theory of Relativity" (Prentice-Hall, New York, 1942), p. 206 ff. for original references and a short exposition.
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16. Egge and Wheeler, *Phys. Rev.* 103, 1063 (1957). Errors in these computations have been pointed out by Marrose, *J. Math. Phys.* 4, 746 (1963).
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18. Brill and Lindquist, *Phys. Rev.* 131, 471 (1963). Fig. 4.